

Spontaneous CP Violation and Natural Flavor Conservation In the $SU(2)_L \times U(1)_Y$ Gauge Theory with Two Higgs Doublets

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Abstract

The two principles of spontaneous CP violation and natural flavor conservation can be realized in the $SU(2)_L \times U(1)_Y$ model with two Higgs doublets. In particular, this model provides a consistent application to the CP violating parameters ϵ and ϵ'/ϵ in kaon decay and the neutron electric dipole moment. The masses of the exotic scalars are unconstrained in this model and probing these exotic scalars must be valuable at the present energy scales. Large CP violations may also occur in the heavy quark and lepton sectors.

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I find that the $SU(2)_L \times U(1)_Y$ gauge theory with two Higgs doublets, which is the simplest extension of the standard $SU(2)_L \times U(1)_Y$ model [1], can realize the two principles that CP conservation is a fundamental symmetry of lagrangian and broken spontaneously, and the neutral currents conserve all flavors naturally, i.e., the usually so-called spontaneous CP violation (SCPV) which was first pointed out by T.D. Lee [2], and the natural flavor conservation (NFC) which was first suggested by Glashow and Weinberg [3], and Paschos [4]. As a consequence, the indirect CP violation (ϵ) [5] and direct CP violation (ϵ'/ϵ) [6] observed in kaon decay and the neutron electric dipole moment (EDM)(d_n) [7] can be easily accounted for in this minimal SCPV and NFC model with two Higgs doublets.

The crucial point to achieve such a model is that the Glashow-Weinberg criterion [3] for NFC in the Higgs sector is found to be only sufficient but not necessary. I then deduce a new theorem of matrix algebra, instead of the discrete symmetries imposed by the Glashow-Weinberg criterion, to ensure NFC in the Higgs sector. Unlike the Glashow-Weinberg criterion, the new theorem allows each Higgs doublet couple to all the fermions (leptons and quarks) even if conforming to the NFC, so that the lagrangian does not possess any additional discrete symmetries. Glashow-Weinberg criterion is only one of the special cases.

Without assumming the NFC [8], the model becomes a general SCPV two-Higgs-doublet model [2], and it usually has flavor changing neutral Higgs exchanges (FCNH) which result in $\Delta S = 2$ superweak interaction (SWI) [9] at tree level. Such a SCPV and SWI model with accepting standard constraints on the Higgs mass was first seriously investigated by Liu and Wolfenstein [10]. They chosen rather to have the unnatural heavy scalars [11] in an $SU(2) \times U(1)$ model than to fine-tune the FCNH couplings to sufficiently small so that to meet the restriction on $\Delta S = 2$ given by the $K_L - K_S$ mass difference. Consequently, they found that such a simple model provided the abundant interesting phenomenological applications.

Anyway we believe that the simplest extended $SU(2)_L \times U(1)_Y$ model with two Higgs doublets is an attractive and fascinating model. I now present a detailed description for an alternative two-Higgs-doublet model which has both the SCPV and the NFC. Let us begin

with the general Yukawa interactions

$$L_Y = \bar{q}_L \Gamma_D^a D_R \phi_a + \bar{q}_L \Gamma_U^a U_R \bar{\phi}_a + \bar{l}_L \Gamma_E^a E_R \phi_a + h.c. \quad (1)$$

where q_i , l_i and ϕ_a are $SU(2)_L$ doublet quarks, leptons and Higgs bosons, while U_i , D_i and E_i are $SU(2)_L$ singlets, Γ_F^a (F=U, D, E) are the Yukawa coupling matrices and real by CP invariance. $i = 1, \dots, n$ is a generation label and $a = 1, 2, \dots, N$ is a Higgs doublet label.

To ensure the NFC, the following theorem in the Higgs sector is deduced to replace the Glashow-Weinberg criterion.

Theorem: for the flavor conservation by the neutral currents to be natural in the Higgs sector, or equivalently, the matrices Γ_F^a (F=U, D, E) $a = 1, 2, \dots, N$ can be diagonalized simultaneously by a biunitary or biorthogonal (for real Γ_F^a) transformation if and only if the square $n \times n$ matrices Γ_F^a are represented in terms of the linear combinations of a complete set of $n \times n$ matrices $\{\Omega_F^\alpha, \alpha = 1, 2, \dots, n\}$, i.e

$$\Gamma_F^a = \sum_{\alpha} g_{a\alpha}^F \Omega_F^\alpha \quad (2)$$

where Ω_F^α satisfy the following orthogonal condition

$$\Omega_F^\alpha (\Omega_F^\beta)^\dagger = L_F^\alpha \delta_{\alpha\beta} ; \quad (\Omega_F^\alpha)^\dagger \Omega_F^\beta = R_F^\alpha \delta_{\alpha\beta} . \quad (3)$$

with the conventional normalization $\sum_{\alpha} L_F^\alpha = \sum_{\alpha} R_F^\alpha = 1$.

This theorem implies that the real matrices Γ_F^a can be written into the following structure

$$\Gamma_F^a = \sum_{\alpha=1}^n g_{a\alpha}^F O_L^F \omega^\alpha (O_R^F)^T , \quad a = 1, \dots, N. \quad (4)$$

with $\{\omega^\alpha, \alpha = 1, \dots, n\}$ the set of diagonalized projection matrices $\omega_{ij}^\alpha = \delta_{i\alpha} \delta_{j\alpha}$. $O_{L,R}^F$ are the arbitrary orthogonal matrices and independent of the Higgs doublet label a , the later feature is the crucial point for ensuring the NFC. $g_{a\alpha}^F$ are the arbitrary real Yukawa coupling constants.

The proof of the above theorem is evident from the well-known theorem of the matrix algebra that any matrix can be diagonalized by a biunitary matrix. I prefer not to present a detailed demonstration on this theorem in this short paper.

Consider now the simplest case of two Higgs doublets, i.e., $N = 2$. After the spontaneous symmetry breaking, the neutral components of the Higgs doublets acquire VEV's

$$\langle \phi_1^0 \rangle = v_1 e^{i\delta} / \sqrt{2}, \quad \langle \phi_2^0 \rangle = v_2 / \sqrt{2}. \quad (5)$$

where a special phase basis is chosen so that v_1 and v_2 are real. The mass matrices are given by the term

$$M_F = \sum_a \hat{v}_a \Gamma_F^a = \sum_{\alpha=1}^n (v_1 e^{i\delta} g_{1\alpha}^F + v_2 g_{2\alpha}^F) O_L^F \omega^\alpha (O_R^F)^T \quad (6)$$

Which are the arbitrary complex matrices. The physical basis is defined through the orthogonal transformations and the phase redefinitions of the fermions and scalars.

$$f_L = (O_L^F)^T F_L; \quad f_R = (O_R^F P^f)^T F_R. \quad (7)$$

where P^f is the diagonal matrix with the phase factor, $P_{ij}^f = e^{i\sigma\delta_{f_i}} \delta_{ij}$, with $\sigma = +$, for $f_i = d_i, e_i$, and $\sigma = -$, for $f_i = u_i$. The phases δ_{f_i} are introduced from the definition

$$(\sin \alpha g_{1i}^F e^{i\sigma\delta} + \cos \alpha g_{2i}^F) v \equiv \sqrt{2} m_{f_i} e^{i\sigma\delta_{f_i}} \quad (8)$$

with $v^2 = v_1^2 + v_2^2 = (\sqrt{2} G_F)^{-1}$, $\sin \alpha = v_1/v$ and $\cos \alpha = v_2/v$. Where m_{f_i} are the masses of the physical states f_i . We may call δ_{f_i} the induced phases from the complex VEV's.

In the physical basis, the Yukawa interaction term becomes

$$\begin{aligned} L_Y = & (2\sqrt{2}G_F)^{1/2} \sum_{i,j=1}^3 [\xi_{d_j} \bar{u}_L^i V_{ij} m_{d_j} d_R^j H^+ - \xi_{u_j}^* \bar{d}_L^i V_{ij}^T m_{u_j} u_R^j H^- \\ & + \xi_{e_j} \bar{\nu}_L^i \delta_{ij} m_{e_j} e_R^j H^+ + h.c.] + (\sqrt{2}G_F)^{1/2} \sum_{i=1}^3 \sum_{k=1}^3 [\eta_{u_i}^{(k)*} m_{u_i} \bar{u}_L^i u_R^i \\ & + \eta_{d_i}^{(k)} m_{d_i} \bar{d}_L^i d_R^i + \eta_{e_i}^{(k)} m_{e_i} \bar{e}_L^i e_R^i + h.c.] H_k^0 \end{aligned} \quad (9)$$

with

$$\xi_{f_i} = \frac{2 \sin \delta_{f_i}}{\sin 2\alpha \sin \delta} e^{i(\delta - \delta_{f_i})} - \tan \alpha; \quad \eta_{f_i}^{(k)} = O_{2k} e^{-i\delta_{f_i}} + (O_{1k} + iO_{3k}) \xi_{f_i}. \quad (10)$$

where $V = (O_L^U)^T O_L^D$ is the real Cabbibo-Kobayashi-Maskawa matrix. $H_k^0 = (h, H, A)$ are the three physical neutral scalars and O_{kl} is the 3×3 orthogonal mixing matrix among these three scalars, H plays the role of the Higgs boson in the standard model.

The induced complex fermion-Higgs boson Yukawa couplings are given by

$$g_{f_i} \equiv (2\sqrt{2}G_F)^{1/2}\xi_{f_i}m_{f_i} = g_{1i}^F \arccos \alpha e^{i(\delta-\delta_{f_i})} - (\sqrt{2}G_F)^{1/2}m_{f_i} \tan \alpha \quad (11)$$

Without making additional assumptions, m_{f_i} , V_{ij} , $m_{H_k^0}$, ξ_{f_i} (or g_{f_i} , or δ_{f_i}), m_{H^+} and O_{kl} (or $\eta_{f_i}^{(k)}$) are in general the free parameters. m_{f_i} , V_{ij} , and m_{H^0} already appear in the standard model with one Higgs doublet. The additional parameters ξ_{f_i} and O_{kl} (or $\eta_{f_i}^{(k)}$) should measure the magnitudes which deviate from ones in the standard model, and describe the new physical phenomena.

An important feature of the present model is that the structure of the fermion-Higgs boson Yukawa couplings in eq.(9) distinguishes from one in the Weinberg three-Higgs-doublet model [12] in which the complex Yukawa couplings (corresponding to the ξ_{f_i}) are the same for the fermions with given charge and depend only on the Higgs bosons. This is because the CP violation of the charged-Higgs boson sector in the Weinberg model originates from the complex mass matrix of the charged-Higgs bosons, for which three Higgs doublets are the minimal number required for a nontrivial CP phase in the charged-Higgs boson mixing matrix. Therefore the CP violations occurred in the processes which involve the different flavors of the same charge are correlated strongly each other in the Weinberg model of CP violation. It is this essential reason that why the Weinberg three-Higgs-doublet model suffers from the inconsistencies between the constraints obtained from K-physics and from the neutron EDM as well as B-physics [13,14]. Unlike the Weinberg model of CP violation, the induced CP violating phases δ_{f_i} (or the corresponding complex Yukawa couplings ξ_{f_i}) of the present model arise from the redefinition of the phases of the quarks and leptons after the spontaneous symmetry breaking, and they are all different and can not be rotated away, so that they become nontrivial and observable in the fermion-Higgs boson interactions. The basic reason for such a case is because all the quarks and leptons in the present model can couple to the two Higgs bosons, and receive their contributions to their mass from the two VEV's and the corresponding two Yukawa couplings which are different for all the fermions. Obviously, these induced phases also contribute to the fermion-neutral Higgs

boson couplings $\eta_{f_i}^{(k)}$. Moreover, there are additional CP violating sources which contribute to the $\eta_{f_i}^{(k)}$. The origin of these sources is well-known due to the scalar-pseudoscalar mixing. As a consequence, the present model evades the difficulties encountered in the Weinberg three-Higgs-doublet model. To see this, let us examine this simple model in its phenomenological applications.

Consider first the CP violating parameters ϵ and ϵ'/ϵ in kaon decays. Like the Weinberg three-Higgs-doublet model, but unlike the Kobayashi-Maskawa model [15], the long-distance contribution to ϵ dominates over the short-distance one. The imaginary part of the $K^0 - \bar{K}^0$ mixing mass matrix to which ϵ is proportional mainly receives the contribution from the π , η and η' poles [16]. Using the estimates given in the literature [14] for the hadronic matrix elements, I find that ϵ can be easily accounted for by taking

$$Im(\xi_s \xi_c^* - 0.05 \xi_d^* \xi_c) \frac{1}{m_{H^+}^2} \left(\ln \frac{m_{H^+}^2}{m_c^2} - \frac{3}{2} \right) \simeq 2.7 \times 10^{-2} GeV^{-2} \quad (12)$$

where the experimental value $|\epsilon| = 2.27 \times 10^{-3}$ has been used.

For the ratio ϵ'/ϵ , the calculations evaluated in Ref. [14,17] can be applied to the present model. In fact, the ratio does not depend on the detail of the CP-odd matrix element, so that their evaluations for the ratio ϵ'/ϵ are also valid for the present model. Within the theoretical uncertainties, the ratio is reanalysed recently to be $\epsilon'/\epsilon = (0.4 - 6.0) \times 10^{-3}$ [14], which is comparable to one calculated from the KM model [18] and is also consistent with the present experimental data [6].

Consider now the neutron EDM, d_n . The present experimental limit on d_n is [7] $d_n < 1.2 \times 10^{-25} ecm$. Applying various well-known scenarios for the calculations of d_n to the present model, the following constraints on parameters ξ_{f_i} and $\eta_{f_i}^{(k)}$ are obtained.

In the quark model through charged-Higgs boson exchange

$$Im(\xi_d \xi_c^*) \frac{1}{m_{H^+}^2} \left(\ln \frac{m_{H^+}^2}{m_c^2} - \frac{3}{4} \right) \lesssim 6.3 \times 10^{-2} GeV^{-2} \quad (13)$$

The Weinberg's gluonic operator through charged-Higgs boson exchange [19] leads to

$$Im(\xi_b \xi_t^*) h_{CH}(m_t, m_b, m_H) \lesssim 3.0 \times 10^{-2} \quad (14)$$

where h_{CH} is a function of the quark- and Higgs-mass arising from the integral of the loop. $h_{CH} \leq 1/8$ and $h_{CH} = 1/12$ for $m_b \ll m_t = m_H$. For the later case, one has $Im(\xi_b \xi_t^*) \leq 0.36$.

From the neutral-Higgs boson exchange [13], it requires

$$Im[\eta_t^{(k)*}]^2 h_{NH}(m_t, m_{H_k^0}) \lesssim 0.18 \quad (15)$$

with h_{NH} a similar function as h_{CH} . $h_{NH} \simeq 0.05$ for $m_t = m_H$.

The most dominant contribution to d_n was found from the gluonic chromoelectric dipole moment (CEDM) [20] induced by the Barr-Zee two-loop mechanism [21]. For $m_t \sim m_H$, it puts a constraint

$$Im(\eta_d^{(k)} \eta_t^{(k)*} + 0.5 \eta_u^{(k)*} \eta_t^{(k)*}) \lesssim 0.2 \quad (16)$$

Combining the result obtained from fitting the ϵ and the constraint from accomodating the d_n in the quark model, it is not difficult to find, for the present experimental lower limit of $m_{H^+} \gtrsim 45\text{GeV}$, that

$$Im \xi_s \xi_c^* \gtrsim 10 ; \quad Im \xi_d \xi_c^* \sim 21 \quad (17)$$

which shows that at least one of the Yukawa couplings should be larger than the one in the standard model. A natural solution is $\sin \alpha \ll 1$, i.e., $v_1 \ll v_2$, which implies that the complex fermion-Higgs boson Yukawa couplings g_{f_i} ($f_i = u_i, d_i, e_i$) can, in general, be larger than the ones in the standard model. The above constraints do not exclude the possible large CP violations in the heavy quark system due to the large complex Yukawa couplings.

Finally, I would like to make the following remarks. First, the domain-wall problem is absent from any additional discrete symmetries except the CP symmetry which is encountered by most SCPV models, it is also supposed to ignore this problem in the present paper. Second, the strong CP problem may be simply evaded by using the well-known Peccei-Quinn mechanism [22] realized in a heavy fermion invisible axion scheme [23]. Since this scheme, on the one hand, is one of the simplest schemes and has an advantage that it leaves the above model unchanged except to regard the orthogonal matrix O_{kl} as a 4×4 matrix to

include the small mixings among the additional singlet scalar and the three physical neutral Higgs bosons, and on the other hand it is also free from the axion domain-wall problems. The third, although CP violation solely originates from the spontaneous symmetry breaking, the mechanism to have CP violation to take place in the lagrangian after the spontaneous symmetry breaking can be different in various models. The examples are the T.D. Lee model [2], the Weinberg model [12], the Liu-Wolfenstein model [10] and the model described in this paper. The fourth, a consistent phenomenological application of the present model may imply that the FCNH in a general two-Higgs-doublet model can be made to be very small and even negligible, so that the SWI through the neutral-Higgs boson exchange still keeps its generic features described from the original motivation by Wolfenstein [9], otherwise it may deviate significantly from a generic superweak model due to an unexpected large value of ϵ'/ϵ [10]. Last but not least, the present model leaves the masses of the exotic scalars unconstrained, which strongly indicates that probing these exotic scalars must be valuable at the present energy scale. Large CP violations in the heavy flavor and lepton sectors may also open a window to the experimental detections.

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